

Berechne die Flächen, die die jeweiligen Funktionen mit der x-Achse einschließen, mittels Integration:

$$\int_1^5 3x + 4 \, dx = \frac{3}{2}x^2 + 4x \Big|_1^5 = \left(\frac{3}{2}5^2 + 4 \cdot 5\right) - \left(\frac{3}{2}1^2 + 4\right) = \frac{115}{2} - \frac{11}{2} = 52$$

$$\int_0^\pi \sin x \, dx = -\cos x \Big|_0^\pi = -\cos(\pi) + \cos(0) = 1 + 1 = 2$$

$$\int_{10}^{20} 3 \, dx = 3x \Big|_{10}^{20} = 3 \cdot 20 - 3 \cdot 10 = 60 - 30 = 30$$

$$\begin{aligned} \int_{90^\circ}^{180^\circ} \sin(x) - \cos(x) \, dx &= (-\cos(x) - \sin(x)) \Big|_{90^\circ}^{180^\circ} \\ &= (-\cos(180^\circ) - \sin(180^\circ)) - (-\cos(90^\circ) - \sin(90^\circ)) \\ &= (1 - 0) - (0 - 1) = 1 + 1 = 2 \end{aligned}$$

$$\begin{aligned} \int_{-1}^1 2x + 1 \, dx &= -\int_{-1}^{-1/2} 2x + 1 \, dx + \int_{-1/2}^1 2x + 1 \, dx \\ &= -(x^2 + x) \Big|_{-1}^{-1/2} + (x^2 + x) \Big|_{-1/2}^1 \\ &= -\left(\left(-\frac{1}{2}\right)^2 - \frac{1}{2} - ((-1)^2 + (-1))\right) + \left(1^2 + 1 - \left(\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)\right)\right) \\ &= +\frac{1}{4} + \frac{9}{4} = \frac{10}{4} \end{aligned}$$

Es darf nicht über eine Nullstelle hinweg integriert werden!

$$\int_1^2 \frac{1}{x} dx = \ln|x| \Big|_1^2 = \ln|2| - \ln|1| = \ln|2| - 0 = \ln|2|$$

$$\int_4^6 4e^x dx = 4e^x \Big|_4^6 = 4(e^6 - e^4)$$

$$\int_0^2 3x^2 + 1 dx = (x^3 + x) \Big|_0^2 = (2^3 + 2) - (0 + 0) = 8 + 2 = 10$$