Löse folgende Gleichungssysteme mit der Cramer'schen Regel

1)

$3x + 2y = 1$

$2x - y = 3$

$$A = \begin{pmatrix} 3 & 2 \\ 2 & -1 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$D = \begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} = -3 - 4 = -7$$

$$D_1 = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -1 - 6 = -7$$

$$D_2 = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 9 - 2 = 7$$

$$L = \left\{ \frac{-7}{-7} \; ; \; \frac{7}{-7} \right\} = \{1 \; ; \; -1\}$$

2)

$4x - y = 0$

$5x + 2y = 1$

$$A = \begin{pmatrix} 4 & -1 \\ 5 & 2 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$D = \begin{vmatrix} 4 & -1 \\ 5 & 2 \end{vmatrix} = 8 - (-5) = 13$$

$$D_1 = \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 0 - (-1) = 1$$

$$D_2 = \begin{vmatrix} 4 & 0 \\ 5 & 1 \end{vmatrix} = 4 - 0 = 4$$

$$L = \left\{ \frac{1}{13} \; ; \; \frac{4}{13} \right\}$$

3)

$$3x - 2y + z = 1$$

$$x + y + z = 10$$

$$2x - 2y - z = 3$$

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 1 & 1 & 1 \\ 2 & -2 & -1 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 10 \\ 3 \end{pmatrix}$$

$$D = \begin{vmatrix} 3 & -2 & 1 \\ 1 & 1 & 1 \\ 2 & -2 & -1 \end{vmatrix} = (-3 - 4 - 2) - (2 - 6 + 2) = -9 + 2 = -7$$

$$D_1 = \begin{vmatrix} 1 & -2 & 1 \\ 10 & 1 & 1 \\ 3 & -2 & -1 \end{vmatrix} = (-1 - 6 - 20) - (3 - 2 + 20) = -27 - 21 = -48$$

$$D_2 = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 10 & 1 \\ 2 & 3 & -1 \end{vmatrix} = (-30 + 2 + 3) - (20 + 9 - 1) = -25 - 28 = -53$$

$$D_3 = \begin{vmatrix} 3 & -2 & 1 \\ 1 & 1 & 10 \\ 2 & -2 & 3 \end{vmatrix} = (9 - 40 - 2) - (2 - 6 - 60) = -33 + 64 = 31$$

$$L = \left\{ \frac{48}{7} \; ; \; \frac{53}{7} \; ; \; \frac{-31}{7} \right\}$$

4)

$$x + y = 1$$

$$2x - z = 1$$

$$y + 3z = 2$$

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{pmatrix} \qquad b = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$D = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \end{vmatrix} = (0 + 0 + 0) - (0 - 1 + 6) = -5$$

$$D_1 = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 2 & 1 & 3 \end{vmatrix} = (0 - 2 + 0) - (0 - 1 + 3) = -2 - 2 = -4$$

$$D_2 = \begin{vmatrix} 1 & 1 & 0 \\ 2 & 1 & -1 \\ 0 & 2 & 3 \end{vmatrix} = (3 + 0 + 0) - (0 - 2 + 6) = 3 - 4 = -1$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix} = (0 + 0 + 2) - (0 + 1 + 4) = 2 - 5 = -3$$

$$L = \left\{ \frac{4}{5} ; \frac{1}{5} ; \frac{3}{5} \right\}$$